PP37085. Proposed by Michaly Bencze.

If $x \in \mathbb{R}$ then $e^{\sin^2 x} + e^{\cos^2 x} \leq 4$.

Solution by Arkady Alt, San Jose, California, USA.

We have $\max\limits_{x \in \mathbb{R}} \left(e^{\sin^2 x} + e^{\cos^2 x}\right) = \max\limits_{0 \leq t \leq 1} (e^t + e^{1-t})$.

Since $(e^t + e^{1-t})' = e^t - e^{1-t} = e^{1-t}(e^{2t-1} - 1)$ and $e^{2t-1} - 1 < 0 \Leftrightarrow t < 1/2$, $e^{2t-1} - 1 > 0 \Leftrightarrow t > 1/2, e^{2t-1} - 1 = 0 \Leftrightarrow t = \dfrac{1}{2}$ then $\min\limits_{x \in \mathbb{R}} \left(e^{\sin^2 x} + e^{\cos^2 x}\right) = \min\limits_{0 \leq t \leq 1} (e^t + e^{1-t}) = 2e^{1/2}$ and $\max\limits_{x \in \mathbb{R}} \left(e^{\sin^2 x} + e^{\cos^2 x}\right) = \max\limits_{0 \leq t \leq 1} (e^t + e^{1-t}) = \max\limits_{t \in \{0,1\}} (e^t + e^{1-t}) = 1 + e$.